2022 Grissom Math Tournament Algebra II Test

1. If the roots of a quadratic equation are real, rational, and unequal, then the discriminant of the equation may be:

   A. -9 B. 0 C. 9 D. 13 E. NOTA

2. Solve the following system of equations: $\begin{matrix} x^2 - xy + y = 5 \\ 2x + y = 3 \end{matrix}$

   A. $\{\left(-\frac{1}{3}, \frac{11}{3}\right), (2, -1)\}$ B. $\{\left(\frac{1}{3}, \frac{8}{3}\right), (-2, 7)\}$ C. $\{(2, 1)\}$
   D. $\{\left(-\frac{1}{3}, \frac{11}{3}\right), (-2, 7)\}$ E. NOTA

3. Find 3 integers that form a geometric progression if their sum is 21 and the sum of their reciprocals is $\frac{7}{12}$.

   A. $\{-3, -6, -12\}$ B. $\{3, 6, 12\}$ C. $\{4, 8, 9\}$ D. $\{-6, -3, -\frac{3}{2}\}$ E. NOTA

4. What is the fourth term in the expansion of $(3x - 2y)^6$?

   A. $4320x^3y^3$ B. $-3240x^3y^3$ C. $3240x^3y^3$ D. $-4320x^3y^3$ E. NOTA

5. Which one of the following is not a function?

   A. $\{(x, y): y = x^3 + 1\}$ B. $\{(x, y): y = x^2 + 1\}$ C. $\{(x, y): x = |y + 1|\}$
   D. $\{(x, y): y = [x] + 1\}$ E. NOTA

6. Find the equation of the circle that passes through the points $A(0, 0); B(4, 0); and\ C(0, -8)$.

   A. $(x - 2)^2 + (y + 4)^2 = 20$ B. $(x + 2)^2 + (y - 4)^2 = 40$
   C. $(x - 4)^2 + (y + 2)^2 = 20$ D. $(x + 4)^2 + (y - 2)^2 = 40$ E. NOTA

7. Find the sum of the arithmetic series: $3 + 9 + 15 + \cdots + 99$.

   A. 812 B. 816 C. 832 D. 846 E. NOTA

8. Given that $\left(1-\sqrt{3}i\right)$ is a solution to the equation $x^4+6x^3-23x^2+54x-44=0$, and the real roots to the equation in simplest form are $m\pm n\sqrt{q}$, find the value of $m+|nq|$.

A. 4 B. 5 C. 6 D. 7 E. 8

9. Solve for n: $\frac{n!}{6!}=\frac{(n-1)!}{4!}$

A. 24 B. 42 C. 36 D. 30 E. NOTA

10. Bowl A contains 3 red chips and 2 white chips. Bowl B contains 4 red chips and 3 blue chips. A chip is drawn at random from bowl A and placed in bowl B. Find the probability of then drawing a red chip from bowl B.

A. $\frac{22}{35}$ B. $\frac{23}{40}$ C. $\frac{9}{16}$ D. $\frac{4}{7}$ E. NOTA

11. Solve: $|x-3|>|2x+1|$

A. $x<\frac{2}{3}$ B. $-4<x<\frac{2}{3}$ C. $x<4$ D. $x<4\ \cup x>\frac{2}{3}$ E. NOTA

12. Find the sum of the lengths of the major and minor axes of the ellipse whose equation is $16x^2+9y^2+64x-18y-71=0$

A. 25 B. 7 C. 14 D. 23 E. NOTA

13. In a survey of 93 people, 25 claimed that they disliked both chocolate and vanilla ice cream. 40 people said they liked chocolate and vanilla. In total, 15 more people liked chocolate than liked vanilla. No one liked vanilla and strawberry while disliking chocolate. 44 liked at least two flavors. If 30 people liked strawberry and 14 liked all three flavors, how many liked none of the flavors?

A. 6 B. 7 C. 12 D. 13 E. NOTA

14. If A = $\begin{bmatrix}5 & 3\\ -7 & -4\end{bmatrix}$ and C = $\begin{bmatrix}39 & -1\\ -53 & 1\end{bmatrix}$, find the sum of the elements of X, if AX=C.

A. 10 B. 2 C. 24 D. 12 E. NOTA

15. A ball is dropped from 10 ft. On each bounce it rebounds to $\frac{4}{5}$ the height from which it fell. How far does the ball travel before coming to rest?

A. 50 ft. B. 100 ft. C. 90 ft. D. 120 ft. E. NOTA

16. If $\log_{21} 7 = a$ and $\log_{21} 6 = b$, find $\log_{21} 2$ in terms of a and b.

A. $a + b - 1$ B. $a + b + 1$ C. $\frac{a+1}{b}$ D. $ab + 1$ E. NOTA

17. Solve: $\frac{x}{x+6} + \frac{2x+3}{2x(x+6)} = \frac{1}{x-1}$

A. $\{-1 \pm \frac{\sqrt{2}}{2}, 3\}$ B. $\{1 \pm \frac{\sqrt{2}}{2}, -3\}$ C. $\{-2 \pm \frac{\sqrt{2}}{2}, -5\}$

D. $\{2 \pm \frac{\sqrt{2}}{2}, 5\}$ E. NOTA

18. If $x = \frac{1-i\sqrt{3}}{2}$, where $i = \sqrt{-1}$, then find $\frac{1}{x^2-x}$.

A. -2 B. -1 C. $1 + i\sqrt{3}$ D. 1 E. NOTA

19. If three, fair six-sided dice are rolled, what is the probability the sum of their face value is 10?

A. $\frac{1}{216}$ B. $\frac{5}{216}$ C. $\frac{5}{108}$ D. $\frac{1}{36}$ E. $\frac{1}{8}$

20. If $g(x) = \frac{x-1}{x+2}$, find $g^{-1}\left(-\frac{3}{2}\right) - g^{-1}\left(\frac{7}{2}\right)$.

A. -4 B. $-\frac{4}{5}$ C. $-\frac{16}{5}$ D. $\frac{12}{5}$ E. NOTA

21. Find the area of the region between the graphs of $|x| + |y| = 1$ and $x^2 + y^2 = 1$.

A. 2 B. π C. 2 - π D. π - 2 E. NOTA

22. Solve for *y*: $\sum_{10}^{13} (yk - 5) = 210$

A. 5 B. 10 C. -5 D. 13 E. NOTA

23. An integer, N, has a remainder of 3 when divided by 4, 6 when divided by 7, and 2 when divided by 3. What is the least possible value of N?

A. 36 B. 41 C. 83 D. 167 E. 419

24. Given $\ln 5 = a,\ \ln 2 = b, \ln 3 = c,\ \ln 11 = d$, which of the following is equivalent to $\ln\left[\left(\frac{5\sqrt{330}}{36}\right)^2\right]$?

A. $3a - 3b - 3c + d$ B. $3a + 3b - 3c + d$ C. $3a + 3b + 3c - d$
D. $3a - b - c + d$ E. $3a - 3b - 3c + 3d$

25. If $f(n) = \frac{1}{3}n(n+1)(n+2)$, then $f(r) - f(r-1) =$

A. $r(r+1)$ B. $(r+2)(r+1)$ C. $\frac{1}{3}r(r+1)$ D. $\frac{1}{3}(r+2)(r+1)$ E. NOTA

Tie Breaker 1: A right triangle has sides 3, 4, and 5. Determine the radius of the inscribed circle.

Tie Breaker 2: How many positive integers between 300 and 500 are divisible by 3 but not 4?

Tie Breaker 3: A treasure is located at a point along a straight road with towns A, B, C, and D on it in that order. A map gives the following instructions for locating the treasure:

a. Start at town A and go $\frac{1}{2}$ of the way to C.
b. Then, from where you are, go $\frac{1}{3}$ of the way to D.
c. Then, go back $\frac{1}{4}$ of the way to B, and dig for the treasure.

If AB = 6 miles, BC = 8 miles, and the treasure is buried midway between A and D, find the distance from C to D.