

2022 Grissom Math Tournament
Comprehensive Test

1. Write in $a + bi$ form: $\frac{6-8i}{2+4i}$
A. $\frac{11}{5} - \frac{2}{5}i$ B. $\frac{11}{5} + \frac{2}{5}i$ C. $\frac{11}{5} + 2i$ D. $-1 + 2i$ E. $-1 - 2i$
2. Solve the following for x : $64^{2x+4} = 256^{x-7}$
A. -20 B. -18 C. 18 D. 20 E. NOTA
3. Sammy the snail travels to his favorite park along a curvy path at a rate of 1 foot per hour. On the trip home, along the same path, he traveled at a rate of 0.75 feet per hour. What is average rate of speed on this trip to the park and back (in feet per hour)?
A. $\frac{5}{6}$ B. $\frac{6}{7}$ C. $\frac{7}{8}$ D. $\frac{8}{9}$ E. $\frac{9}{10}$
4. If the point (a, b) is the solution to the system: $7a + 8b = 23$ and $14a + 3b = 20$, find the value of the expression: $2a - 17b$.
A. -36 B. -32 C. -13 D. 21 E. NOTA
5. Evaluate: $\log_9 16 + \log_{81} 25 - \log_{27} 125 = \log_3 x$
A. $\frac{4\sqrt{5}}{5}$ B. $4\sqrt{5}$ C. 10 D. $10\sqrt{5}$ E. 20
6. Sandy has some cool slimy and scary pets. Her pets are snakes, salamanders, and spiders. In her collection there are 114 heads, 59 tails, and 548 legs. If she trades her spiders for snapping turtles at a rate of 5 spiders are equal to 2 snapping turtles, how many snapping turtles will she have?
A. 20 B. 22 C. 24 D. 26 E. 28
7. Find the sum of the first sixteen perfect squares of positive integers.
A. 1140 B. 1240 C. 1396 D. 1496 E. 1540
8. Evaluate the following if $x = -\frac{7\pi}{6}$: $\frac{\cot x}{1-\csc x}$
A. $-\frac{\sqrt{3}}{3}$ B. $\frac{\sqrt{3}}{3}$ C. $\sqrt{3}$ D. $3\sqrt{3} + 6$ E. $-3\sqrt{3} - 6$

9. Find the value of x^2y^3 if (x, y) is the solution to the system of equations:

$$\frac{6}{x-1} + \frac{9}{y+3} = 24 \quad \text{and} \quad \frac{4}{x-1} - \frac{3}{y+3} = 10$$
- A. -162 B. -54 C. -6 D. $\frac{8}{3}$ E. 6
10. Given three matrices: $A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$, find $AB - AC$.
- A. $\begin{bmatrix} -2 & 10 \\ -9 & 12 \end{bmatrix}$ B. $\begin{bmatrix} 8 & 19 \\ 3 & 18 \end{bmatrix}$ C. $\begin{bmatrix} 8 & 19 \\ -2 & 13 \end{bmatrix}$ D. $\begin{bmatrix} -17 & 19 \\ -15 & 18 \end{bmatrix}$ E. NOTA
11. Solve for x : $\log_2(7 - x) = 3 + \log_2(2x + 3)$.
- A. $-\frac{20}{19}$ B. -2 C. -1 D. $-\frac{11}{13}$ E. NOTA
12. Evaluate: $\sum_{n=2}^{\infty} 40 \cdot \left(\frac{1}{5}\right)^n = ?$
- A. 2 B. 10 C. 50 D. 200 E. NOTA
13. Evaluate: $\left(\cos \frac{7\pi}{18}\right)\left(\sin \frac{\pi}{18}\right) - \left(\cos \frac{\pi}{9}\right)\left(\cos \frac{\pi}{18}\right)$
- A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{1}{2}$ C. $\frac{\sqrt{6}-\sqrt{2}}{4}$ D. $\frac{-\sqrt{6}+\sqrt{2}}{4}$ E. NOTA
14. What is the remainder when 17^{2022} is divided by 13?
- A. 2 B. 4 C. 6 D. 8 E. NOTA
15. If $z_1 = 12(\cos 120^\circ + i \sin 120^\circ)$, $z_2 = 6(\cos 110^\circ + i \sin 110^\circ)$, and $z_3 = 2(\cos 40^\circ + i \sin 40^\circ)$, write the following expression in $a + bi$ form: $\frac{(z_1) \cdot (z_2)}{(z_3)^2}$
- A. $-9 + 9\sqrt{3}i$ B. $-9\sqrt{3} + 9i$ C. $-18 - 18\sqrt{3}$ D. $36 - 36\sqrt{3}$ E. $-36 + 36\sqrt{3}$
16. Seven points are equally spaced around the circumference of a circle. How many distinct acute triangles can be drawn with these points as vertices?
- A. 6 B. 7 C. 14 D. 21 E. NOTA

17. If the equations of the asymptotes of the graph of: $\frac{(x-3)^2}{8} - \frac{(y+5)^2}{18} = 1$ are written in the form: $y = m \pm n(x - p)$, find the value of $\frac{mn^2}{p}$.

- A. $-\frac{135}{16}$ B. $\frac{27}{20}$ C. $\frac{15}{4}$ D. $\frac{9}{4}$ E. $-\frac{15}{4}$

18. Solve for x: $\ln 729 \div \frac{\ln 8}{\ln 2} = \ln x \div \frac{\ln 25}{\ln 5}$

- A. 3 B. 9 C. 27 D. 81 E. NOTA

19. If $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$, find $A^{-1} \cdot \begin{bmatrix} 10 \\ 10 \\ 20 \end{bmatrix}$.

- A. $\begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$ B. $\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ C. $\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$ D. $\begin{bmatrix} -5 \\ 0 \\ 5 \end{bmatrix}$ E. NOTA

20. What is the smallest integer with remainder 2 when divided by 7 and 9 and remainder 6 when divided by 8?

- A. 65 B. 78 C. 128 D. 254 E. NOTA

21. Evaluate: $\sum_{n=6}^{\infty} \left(\frac{1}{n^2 - n - 2} \right)$

- A. $\frac{1}{12}$ B. $\frac{3}{20}$ C. $\frac{9}{20}$ D. $\frac{37}{60}$ E. NOTA

22. What is the volume of tetrahedron NERD with vertices at N (1, 0, 2), E (2, 1, 4), R (3, 0, 5), and D (1, 5, 1)?

- A. $\frac{1}{2}$ B. $\frac{5}{6}$ C. $\frac{7}{6}$ D. $\frac{7}{3}$ E. NOTA

23. Given the function: $f(x) = 12x^3 - 16x^2 - 73x + 105$, what is the sum of the reciprocals of the roots of $f(x)$?

- A. $\frac{4}{3}$ B. $\frac{3}{4}$ C. $\frac{105}{73}$ D. $\frac{73}{105}$ E. NOTA

24. Triangle ABC is a right triangle with right angle at B. Circle P is the inscribed circle of triangle ABC, and circle M is inside the triangle tangent to circle P and the two sides \overline{AB} and \overline{BC} . Find the ratio of the area of circle P to the area of circle M.

- A. $17 - 12\sqrt{2}$ B. $17 + 12\sqrt{2}$ C. $3 + 2\sqrt{2}$ D. $3 + 2\sqrt{2}$ E. NOTA

25. For all positive integers $n > 10$, such that $(n^3 - 4n + 5)$ is not divisible by 2, what is the largest integer value of k such that $(n^4 - 2n^3 - 4n^2 + 8n)$ must be divisible by 2^k ?
- A. 4 B. 5 C. 6 D. 7 E. NOTA

Tie Breaker 1: How many odd integers between 100 and 700 are divisible by 3 and not 5?

Tie Breaker 2: Find the sum of the three largest three-digit prime numbers.

Tie Breaker 3: A page number in a book is erased. The sum of all the remaining page numbers in the book is 3783. How many page numbers are in the book?