Rocket City Math League

Interschool Test

2021-2022
Senior

1. **Answer: $16,954$**
   $276\frac{4}{5} \times 61.25 = 16{,}954$

2. **Answer: $y = \frac{x-1}{2x+3}$**

   $10y - 5 + \frac{11y+7}{x} - \frac{6y}{x^2} = \frac{2}{x^2}$ multiply by x^2
   $10yx^2 - 5x^2 + 11xy + 7x - 6y = 2$
   $y(10x^2 + 11x - 6) = 5x^2 - 7x + 2$
   $y = \frac{5x^2-7x+2}{10x^2+11x-6} = \frac{(5x-2)(x-1)}{(5x-2)(2x+3)}$

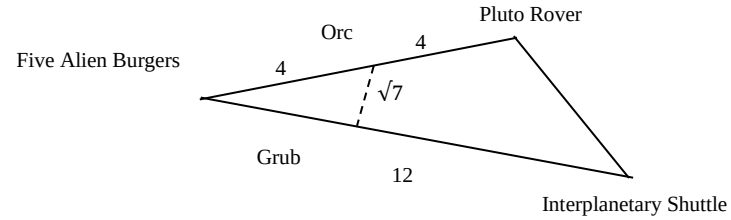
3. **Answer: $10,080$**
   "ABSTRACT" has 8 letters and 2 pairs of repeated letters. Due to this, the equation to solve this problem is $\frac{8!}{2!\cdot 2!}$ which equals 10,080.

4. **Answer: 48**
   If Bob wears white socks, then there are 4 shirts, 3 pants, and 3 shoes that he can chose from. This means that with white socks Bob can wear 36 outfits. If Bob wears black socks, then there are 4 shirts, 1 pair pants, and 3 shoes that he can chose from. This means that with black socks Bob can wear 12 outfits. The total number of outfits that Bob can wear is 48.

5. **Answer: $90\frac{20}{27}$ or $\frac{2450}{27}$ ft**
   The equation for the volume of the box formed is $y = x(8-2x)(15-2x)$ where x is the length of the squares that have been cut from the piece of carbon fiber and the y is the volume of the box. The y-value of the maximum of the graph will give you the maximum volume of the box. The Maximum of the graph will occur where the values of the function's derivative change from positive to negative due to the first derivative test.
   $f(x) = x(8-2x)(15-2x) = 4x^3 - 46x^2 + 120x$
   $f'(x) = 12x^2 - 92x + 120 = (3x-5)(x-6)$
   Because the shorter side is 8ft long Fridz cannot cut more than 4ft, so 5/3 is the only root that must work. Plugging this value back into the original function will give the value of the maximum.
   $f\left(\frac{5}{3}\right) = 4\left(\frac{5}{3}\right)^3 - 46\left(\frac{5}{3}\right)^2 + 120\left(\frac{5}{3}\right) = 90\frac{20}{27} = \frac{2450}{27}$

6. **Answer: 364**
   $\left(x - \frac{1}{x}\right)^6 = x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$
   $(x+3)^5 = x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$
   $(x+2)^4 = x^4 + 8x^3 + 24x^2 + 32x + 16$
   $(x+5)^3 = x^3 + 15x^2 + 75x + 125$
   $-20 + 243 + 16 + 125 = 364$

7. **Answer: 8 km**
   Using the information provided in the question, make a triangle with vertices at Five Alien Burgers, the Pluto Rover, and the Interplanetary Shuttle.



   Use the distance between Orc and Five Alien Burgers and between Orc and Grub to calculate the angle between their paths.
   $\sin\theta = \frac{\sqrt{7}}{4}$ $\quad \sin^{-1}\frac{\sqrt{7}}{4} = \theta$ $\quad cos\theta = \sqrt{1 - 7/16} = \frac{3}{4}$
   Now use the law of cosines to calculate the distance between the Pluto Rover and the Interplanetary Shuttle, the side of the bigger triangle opposite the angle between Orc and Grub's paths.
   $c^2 = a^2 + b^2 - 2ab \cdot \cos C$ $\quad c^2 = 8^2 + 12^2 - 2(8)(12)\left(\frac{3}{4}\right)$ $\quad$ c = 8

8. **Answer: $\frac{196\pi \pm 49\sqrt{3} - 98}{4}$**
   Calculate the area of the circle with radius 7 using the are of circle formula
   $A = \pi r^2$ $\quad A = 7^2 \cdot \pi = 49\pi$
   There are two ways to draw the described inscribed triangle: an obtuse isosceles triangle (figure 8a) and an acute isosceles triangle (figure b).

Figure 8a

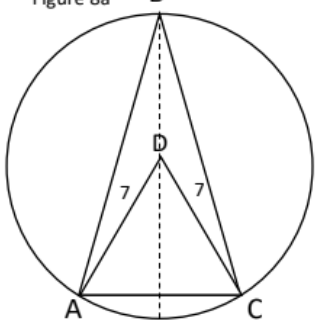
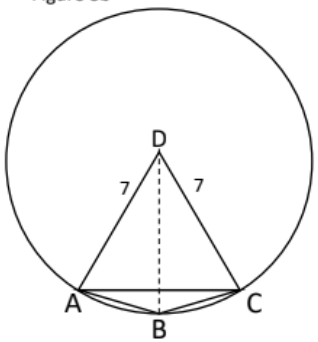


Figure 8b



In both cases triangle ACD is an equilateral triangle and triangles ABD and BCD are congruent isosceles triangles. In the case of figure 8a, the area of triangles ABD + BCD +ACD= the area of triangle ABC.

$2\left(\frac{1}{2}\cdot 7^2\cdot \sin 150°\right)+\left(\frac{1}{2}\cdot 7^2\cdot \sin 60°\right)=\frac{98+49\sqrt{3}}{4}$

In the case of figure 8b, the area of triangles ABD + BCD – ACD = the area of triangle ABC.

$2\left(\frac{1}{2}\cdot 7^2\cdot \sin 30°\right)-\left(\frac{1}{2}\cdot 7^2\cdot \sin 60°\right)=\frac{98-49\sqrt{3}}{4}$

Subtract these areas from the area of the circle to find the area of the circle that is outside the inscribed triangle.

$49\pi-\frac{98\pm 49\sqrt{3}}{4}=\frac{196\pi\pm 49\sqrt{3}-98}{4}$

9. **Answer: $\$4.22$**

The function $0.01p+0.05n+0.1d+0.25q=M$ represents the amount of money that Redgy has. Using Gauss Jordan Elimination to solve the system of equations, you find that $p=22, n=13, d=16, and\ q=7$. Plugging in these values gives you that Redgy has $4.22.

10. **Answer: $\boldsymbol{8x^2+7x+36+\frac{194x+60}{x^2-5x-2}}$**

Using long division, the quotient and remainder are:

$$\frac{8x^4-33x^3-15x^2-12}{x^2-5x-2}=\boldsymbol{8x^2+7x+36+\frac{194x+60}{x^2-5x-2}}$$

11.

Answer: $\frac{5}{2}$

The Volume of a tetrahedron with vertices at M, A, T, and H can be found using the formula $V=\frac{1}{6}\left[\overrightarrow{MA}\cdot(\overrightarrow{MT}\cdot\overrightarrow{MH})\right]$.

$\overrightarrow{MA}=\langle 0,2,1\rangle$ $\quad\overrightarrow{MT}=\langle 2,1,2\rangle$ $\quad\overrightarrow{MA}=\langle 1,3,6\rangle$

$$\frac{1}{6}\begin{vmatrix}0&2&1\\2&1&2\\1&3&6\end{vmatrix}=\frac{1}{6}|(0+4+6)-(1+0+24)|=\frac{1}{6}|10-25|=\frac{15}{6}=\frac{5}{2}$$

11. **Answer: 30.5%**

A = 70	B = 100	C = 80	D = 60
AB = 40	AC = 31	AD = 28	BC = 48
BD = 24	CD = 31	ABC = 25	ABD = y
ACD = z	BCD = 22	ABCD = x	none = 20

$(A\cap B\cap C\cap D) = x$
$(A\cap B\cap C^c\cap D) = y - x$
$(A^c\cap B\cap C\cap D) = 22 - x$
$(A\cap B^c\cap C\cap D^c) = 6 + x - z$
$(A^c\cap B\cap C\cap D^c) = 1 + x$
$(A^c\cap B^c\cap C\cap D) = 9 + x - z$
$(A^c\cap B\cap C^c\cap D^c) = 35 - x + y$
$(A^c\cap B^c\cap C^c\cap D) = -1 - x + y + z$
$(A\cap B\cap C\cap D^c) = 25 - x$
$(A\cap B^c\cap C\cap D) = z - x$
$(A\cap B\cap C^c\cap D^c) = 15 + x - y$
$(A\cap B^c\cap C^c\cap D) = 28 + x - y - z$
$(A^c\cap B\cap C^c\cap D) = 2 + x - y$
$(A\cap B^c\cap C^c\cap D^c) = -4 - x + y + z$
$(A^c\cap B^c\cap C\cap D^c) = 17 - x + z$

We know that 19%, or 38, of the students liked at least 3 fruits and that 90%, or 180, of the students likes at least one. Using these values and the equations created above create a system of equations.

$$\begin{cases} 3x - y - z = 9 \\ -x + y + z = 25 \end{cases} \quad = \quad 2x = 34 \quad \text{so } x = 17$$

$(A^c \cap B \cap C \cap D) = 22 - x = 5$ $\qquad$ $(A \cap B \cap C \cap D^c) = 25 - x = 8$

$BD = (A \cap B \cap C \cap D) + (A^c \cap B \cap C \cap D) + (A \cap B \cap C^c \cap D)$

$24 = 17 + 5 + y - 17 \qquad y = 19$

$AC = (A \cap B \cap C \cap D) + (A \cap B \cap C \cap D^c) + (A \cap B^c \cap C \cap D)$

$31 = 17 + 8 + z - 17 \qquad z = 23$

Now find the number of students who just like bananas and just like dragon fruit.

$(A^c \cap B \cap C^c \cap D^c) = 35 - x + y = 37$ $\qquad$ $(A^c \cap B^c \cap C^c \cap D) = -1 - x + y + z = 24$

The probability that students only like bananas or only like dragon fruit is $\frac{37+24}{200} = 0.305$

12. **Answer: -12**

$\log_a m = 4 \qquad \log_b m = 6 \qquad \log_{\frac{ab}{c}} m = 2$

$\log_{\frac{ab}{c}} m = \frac{\log m}{\log \frac{ab}{c}} = \frac{\log m}{\log a + \log b - \log c} = 2$

$\frac{1}{2} = \frac{\log a + \log b - \log c}{\log m} = \frac{\log a}{\log m} + \frac{\log b}{\log m} - \frac{\log c}{\log m} = \log_m a + \log_m b - \log_m c$

$\frac{1}{2} = \log_m a + \log_m b - \log_m c = \frac{1}{4} + \frac{1}{6} - \log_m c$

$\log_m c = \frac{1}{4} + \frac{1}{6} - \frac{1}{2} = \frac{-1}{12}$ $\qquad$ so $\qquad$ $\log_c m = -12$

13. **Answer: -1**

$\frac{2\tan\theta\cos^2\theta}{\cos 2\theta} = \frac{2\tan\theta\cos^2\theta}{2\cos^2\theta - 1}$ because $\cos 2\theta = 2\cos^2\theta - 1$ (double-angle formula)

$\frac{2\tan\theta\cos^2\theta}{2\cos^2\theta - 1} = \frac{2\tan\theta}{2 - \sec^2\theta}$ by dividing by cosine

$\frac{2\tan\theta}{2 - \sec^2\theta} = \frac{2\tan\theta}{1 - (\sec^2\theta - 1)} = \frac{2\tan\theta}{1 - \tan^2\theta}$ because $\sec^2\theta - 1 = \tan^2\theta$ (Pythagorean identity)

$\frac{2\tan\theta}{1 - \tan^2\theta} = \tan 2\theta$ (double-angle formula)

When $\theta = \frac{7\pi}{8}$, $\tan 2\theta = -1$

14. **Answer: 229**

$\frac{N}{270} = 0.a1a51a51a51\overline{a5}$

$x = 0.a1a51a51a51\overline{a5}$

$10x = a.1a51a51a51\overline{a5}$

$10{,}000x = a,1a5.1a51a51a51\overline{a5}$

$10{,}000x - 10x = 9{,}990x = a,1a5 - a$

$x = \frac{a,1a5 - a}{9{,}990} = \frac{N}{270}$

$N = \frac{a,1a5 - a}{37}$

Now plug in single digit values for a and see which leaves the expression as an integer.

a=	$N = \frac{a,1a5-a}{37}$
0	$\frac{105}{37} = 2.8\ldots$
1	$\frac{1115-1}{37} = 30.1\ldots$
2	$\frac{2125-2}{37} = 57.3\ldots$
3	$\frac{3135-3}{37} = 84.6\ldots$
4	$\frac{4145-4}{37} = 111.9\ldots$
5	$\frac{5155-5}{37} = 139.1\ldots$
6	$\frac{6165-6}{37} = 166.4\ldots$
7	$\frac{7175-7}{37} = 193.7\ldots$
8	$\frac{8185-8}{37} = 221$

So a=8 and N=221 so a+N=229