# Rocket City Math League

**2021-2022**

## Interschool Test

**Senior**

Write the answers on the answer sheet provided. All answers must be written in exact, reduced, simplified, and rationalized form. Leave answers in terms of $\boldsymbol{\pi}$ unless otherwise specified. **No calculators, books, or other aids may be used.** **Time: 30 minutes**

1. Multiply $276\frac{4}{5}$ by $61.25$ *(3 points)*

2. Solve for y in terms of x: $10y - 5 + \frac{11y+7}{x} - \frac{6y}{x^2} = \frac{2}{x^2}$ where $x > 1$ *(3 points)*

3. How many distinct arrangements are there of the letters in the word ABSTRACT? *(3 points)*

4. Bob has a green shirt, a red shirt, a black shirt, and a white shirt. He also has blue pants, red pants, and black pants. He also has a pair of white socks and black socks. He also has boots, tennis shoes, and vans. If Bob never wears black socks unless he is wearing his black pants, how many distinct outfits can Bob wear? *(3 points)*

5. Fridz has an 8ft by 15ft piece of carbon fiber. He cuts four congruent squares out of the corners of the sheet of material and folds up the sides to make an open top rectangular box. Find the maximum volume of the box in cubic feet. *(3 points)*

6. What is the constant term when the following is expanded and like terms combined?
$\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right)^6 + (\mathrm{x}+3)^5 + (\mathrm{x}+2)^4 + (2\mathrm{x}+5)^3$ *(4 points)*

7. Grub and Orc had a snack at Five Aliens Burgers. When they left, Grub followed a linear path to his Interplanetary Shuttle service 12 kilometers away, and Orc followed a different linear path to his Pluto Rover which was 8 kilometers away. When Orc was half of the way to his Pluto Rover, he noticed that the distance from his location to Grub's path was $\sqrt{7}$ kilometers. When both have reached their destinations, how far apart are they (in kilometers)? *(4 points)*

8. An isosceles triangle is inscribed in a circle. The circle has a radius of 7 and the base of the triangle is also 7. Find the area of the part of the circle that is outside of the triangle. *(4 points)*

9. Redgy has a certain number of pennies (p), nickels (n) dimes (d), and quarters (q). Given the following equations, how much money does Redgy have?

$$\begin{cases} p + q = n + d \\ p + n + d + q = 58 \\ p + d + 2q = 4n \\ p + 2n + 1.5d + 4q = 100 \end{cases}$$

*(4 points)*

10. Divide $8x^4 - 33x^3 - 15x^2 - 12$ by $x^2 - 5x - 2$ *(4 points)*

11. What is the volume of the tetrahedron with vertices M(0,2,5), A(0,4,6), T(2,3,7), and H(1,5,11)? *(5 points)*

12. 200 students at our high school were asked which fruits they liked. Their choices were apples, bananas, cantaloupe, and dragon fruit. 90% liked at least one of these and 19% liked at least three.
70 students liked apples
60 students like dragon fruit
40% of students like cantaloupe
50% of students like bananas
25 students like apples, bananas, and cantaloupe
22 students like bananas, cantaloupe, and dragon fruit
31 students like apples and cantaloupe
24 students like bananas and dragon fruit
40 students like apples and bananas
48 students like bananas and cantaloupe
31 students like cantaloupe and dragon fruit
28 students like apples and dragon fruit
What percentage of students like only dragon fruit or only bananas? *(5 points)*

13. Given that $a, b, c > 1$ and $m > 0$

$$\log_a m = 4$$
$$\log_b m = 6$$
$$\log_{\frac{ab}{c}} m = 2$$

Find $\log_c m$

*(5 points)*

14. Simplify:
$\frac{2\tan\theta\cos^2\theta}{\cos 2\theta}$ if $\theta = \frac{7\pi}{8}$

*(5 points)*

15. Suppose $N$ is a positive integer and $a$ is a single digit in base 10. Find the value of $N + a$ if:
$\frac{N}{270} = 0.a1a51a51a51a5\ldots$

*(5 points)*

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