Grissom Math Tournament
Algebra II Test
January 13, 2007

1. Which of the following has the greatest value?

   A. $\frac{7}{13}$ B. $\frac{10}{19}$ C. $\frac{5}{9}$ D. $\frac{14}{27}$ E. $\frac{15}{29}$

2. Evaluate: $(-8)^{-\frac{2}{3}} + (-8)^{-\frac{1}{3}} + (-8)^{0} + (-8)^{\frac{1}{3}} + (-8)^{\frac{2}{3}}$

   A. $-\frac{5}{4}$ B. $\frac{3}{4}$ C. $\frac{5}{4}$ D. $\frac{9}{4}$ E. $\frac{11}{4}$

3. Solve the following for x: $3[2 - 5(x + 1) + x] = -7(2x + 3) - 4$

   A. -23 B. -8 C. -2.6 D. 4 E. 16

4. Evaluate if x = 2: $\dfrac{x + \frac{1}{x-1}}{x - \frac{1}{x+1}}$.

   A. $\frac{1}{5}$ B. 2 C. $\frac{9}{5}$ D. $\frac{9}{2}$ E. undefined

5. Find the coordinates of the x-intercept of the line perpendicular to $2x - 3y = 8$ and passing through the point (4, 3).

   A. (4, 0) B. (6, 0) C. (9, 0) D. (0, 6) E. (0, 9)

6. If the length of the base of a triangle is increased by 20% and its height is reduced by 25%, which of the following describes the change in its area?

   A. dec. by 5% B. inc. by 5% C. dec. by 10% D. inc. by 10% E. none of these

7. Find the conjugate of $\frac{10}{3+4i}$ in a + bi form.

   A. $\frac{10}{3-4i}$ B. $\frac{10}{3} - \frac{5}{2}i$ C. $\frac{6}{5} - \frac{8}{5}i$ D. $\frac{6}{5} + \frac{8}{5}i$ E. $6 - 8i$

8. Simplify: $i^9 + i^{18} + i^{27} + i^{36} + ... + i^{2007}$.

   A. i B. -1 C. $-i$ D. $-1 + i$ E. 0

9. Solve for x: $\begin{vmatrix} 3 & x-5 \\ 2 & x+1 \end{vmatrix} = 7$

A. -6 B. $-\frac{6}{5}$ C. 0 D. $\frac{11}{5}$ E. $\frac{16}{5}$

10. How many gallons of water must be evaporated from 70 gallons of a 20% salt solution to leave a 35% salt solution?

A. 25 B. 30 C. 35 D. 40 E. 45

11. For what value(s) of k is (x + 3) a factor of $2x^3 - 5x^2 + kx + 27$?

A. -36 B. -24 C. -20 D. 24 E. 42

12. Find $4x - y$ given: $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$.

A. -6 B. -4 C. 0 D. $\frac{3}{2}$ E. 4

13. Solve for x: $(x^2 - 4)(x^2 - 5x + 6) < 0$.

A. $(-\infty, -2)$ B. (-2, 3) C. $(-2,-1) \cup (2,6)$ D. (-1, 2) E. none of these

14. If the quadratic function passing through the points (3, -3), (2, -9), and (5, -3) is written in the form $f(x) = ax^2 + bx + c$, then find the value of ac.

A. -30 B. -4 C. 2 D. 66 E. 252

15. Find the largest value of x satisfying the equation: $\left(\frac{x-3}{2x+1}\right)^2 = 5\left(\frac{x-3}{2x+1}\right) + 6$.

A. -4 B. $-\frac{5}{3}$ C. $-\frac{9}{11}$ D. $\frac{2}{3}$ E. 65

16. Find the area of the region bounded by the graph of $|x + y| \leq 3$.

A. 3 B. 6 C. 9 D. 12 E. 18

17. What is the remainder when (1! + 2! + 3! + 4! + . . . + 2006! + 2007!) is divided by 7?

A. 6 B. 5 C. 4 D. 3 E. 2

18. If $f(x) = 2x^3 + 7x^2 + 2x$, then find $f^{-1}(-7)$.

A. $-\frac{7}{2}$ B. $-\frac{3}{2}$ C. ± 1 D. $\frac{3}{2}$ E. undefined

19. If i is one of the solutions to $x^4 - 4x^3 + 2x^2 - 4x + 1 = 0$, then find the sum of the squares of the real roots.

A. 18 B. 16 C. 14 D. 4 E. 0

20. Given: $x = y + 8$, $y = z - 6$, and $z = 2x + y$, find the value of z.

A. 1 B. 2 C. 3 D. 4 E. 5

21. Sam has 48 feet of fence to enclose a rectangular pen along the back of her house. (The house will form one of the four sides of the rectangle.) She wants the pen to have the maximum possible area. If the dimensions of the pen are m x n and its area is A, find m + n + A.

A. 288 B. 324 C. 336 D. 520 E. 624

22. Find the value of the sum: $\sum_{k=1}^{2007} (-1)^k k$. (Note, $\sum_{k=1}^{2007} (-1)^k k$ = -1 + 2 + -3 . . . + -2007.)

A. -1004 B. -1003 C. -1002 D. 1003 E. 1004

23. Given that a, b, and c form a set of consecutive multiples of 4 such that the product of the two largest minus the product of the two smallest is 256, find ac.

A. 324 B. 768 C. 896 D. 908 E. 1008

24. Two distinct integers are chosen at random from the interval [1,25]. Find the probability their sum is even.

A. $\frac{288}{625}$ B. $\frac{12}{25}$ C. $\frac{1}{2}$ D. $\frac{13}{25}$ E. $\frac{337}{625}$

25. Find $|x+y+z|$ given the following:

$$(x+y)(x+z) = 6$$
$$(x+y)(y+z) = 30$$
$$(y+z)(x+z) = 45$$

A. 10 B. 18 C. 20 D. 24 E. 27

TB1: Find n if $n = (\log_2 25)(\log_5 243)(\log_3 16)$

TB2: How many integers less than 2007 are multiples of 3 but not multiples of 5?

T B3: If 8 women take 12 days to assemble 32 machines, how many days will it take 18 women to assemble 90 machines? (Assuming all women work at the same constant rate.)