Grissom Math Tournament
Comprehensive Test
January 13, 2007

1. Which of the following has the greatest value?

A. $\frac{7}{13}$ B. $\frac{10}{19}$ C. $\frac{5}{9}$ D. $\frac{14}{27}$ E. $\frac{15}{29}$

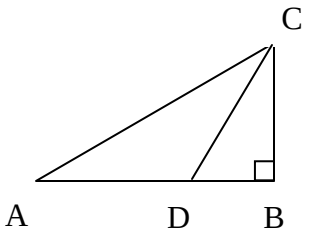
2. If $n = x - y^{(x-y)}$, then find n if x = 2 and y = -2.

A. -14 B. 0 C. 1 D. 4 E. 18

3. A regular polygon has n sides and 2n diagonals. How many sides does the polygon have?

A. 4 B. 5 C. 6 D. 7 E. 8

4. Given right triangle ABC with point D lying on side AB such that BD = 5, AC = 20, and BC = 12, find CD – AD.



A. -2 B. 2 C. 5 D. 11 E. 12

5. If $\frac{a}{b} = \frac{3}{2}$, then evaluate: $\frac{3a^2 - 4b^2}{4a^2 + 3b^2}$.

A. $\frac{11}{48}$ B. $\frac{23}{12}$ C. $\frac{11}{3}$ D. 33 E. cannot be determined

6. If three vertices of a square are (1, 3), (4, 7), and (8, 4), and if (m, n) is the point where the diagonals intersect, then find m + n.

A. 4 B. 5 C. 6 D. 7 E. 8

7. Given $\log_5 2 = a$ and $\log_3 5 = b$, write $\log_5 216$ in terms of a and b.

A. $\left(a + \frac{1}{b}\right)^3$ B. $3a + \frac{3}{b}$ C. $\frac{a^3}{b^3}$ D. 3ab E. a^3b^3

8. How many integers are solutions to the inequality: $\left|\frac{2+x}{7-x}\right| \geq 5$?

A. 3 B. 4 C. 5 D. 6 E. ∞ many

9. Given the function: $f(x) = \frac{2x+5}{x+k}$ where k is a real number, find the value of k so that f(x) is its own inverse function.

A. -5 B. -2 C. $-\frac{5}{2}$ D. 2 E. $\frac{5}{2}$

10. If the fraction $\frac{3}{7}$ is expressed as a repeating decimal, find the 2007th digit to the right of the decimal point.

A. 2 B. 4 C. 5 D. 7 E. 8

11. Find the product of all real values of t that satisfy the equation: $8\left(2^{3t}\right) - 49\left(2^{2t}\right) + 70\left(2^{t}\right) - 8 = 0.$

A. -6 B. $-\frac{3}{2}$ C. 2 D. 3 E. 6

12. Which of the following is closest to the largest solution to the equation: $\begin{vmatrix} 1 & 3 & 1 \\ -x & -2 & 2 \\ -\frac{1}{6} & 0 & x \end{vmatrix} = 0$?

A. 0.5 B. 1.0 C. 1.5 D. 2.0 E. 2.5

13. Solve the following for x: $\log_3 27 - \log_9 27 - \log_{27} 9 = \log_3 x$.

A. $\sqrt[3]{9}$ B. $\sqrt[4]{27}$ C. $\sqrt[6]{243}$ D. $3\sqrt[6]{3}$ E. $9\sqrt[6]{3}$

14. Evaluate: $\frac{\sin 70^\circ \sin 20^\circ}{\cos 40^\circ \cot 50^\circ}$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. 2

15. A line segment with endpoints A(4, -2) and B(8, 1) is extended through A to point C (x, y). If BC = 3(AB), find x – y.

A. 3 B. 4 C. 5 D. 6 E. 7

16. Cash prizes are to be awarded to the top scorers at NCLB High School in 2007. The amounts of the awards are \$1, \$7, \$49, and \$343. What is the least number of awards that can be given if exactly \$2007 will be awarded?

A. 17 B. 19 C. 21 D. 23 E. 25

17. A triangle is formed with vertices at the vertex and the endpoints of the latus rectum of the parabola with equation $x^2 - 6x - 16y + 89 = 0$. Find the ratio of the perimeter to the area of this triangle.

A. $\frac{1+\sqrt{5}}{2}$ B. $\frac{2+\sqrt{5}}{2}$ C. $\frac{1+\sqrt{5}}{4}$ D. $\frac{2+\sqrt{5}}{4}$ E. none of these

18. A speedboat is about to cross a channel 28 miles wide. The channel flows westward with a current of 18 mph. The boat leaves the south bank of the channel at 2:00 and lands on the north bank at a point due north of his starting point. Assuming the boat took a straight line course and maintained a constant speed of 30 mph, at what time does the boat land?

A. 2:30 B. 2:40 C. 2:50 D. 3:00 E. 3:10

19. Find the product of the roots of $4\log_3 x - 8 = 5\log_x 3$.

A. 1 B. 3 C. $3\sqrt{3}$ D. 9 E. $9\sqrt{3}$

20. How many pairs of positive integers (x, y) satisfy the equation: $x^2 + 2007 = y^2$?

A. 1 B. 2 C. 3 D. 4 E. 5

21. Two roots of $4x^3 + ax^2 + bx - 4 = 0$ are $-\frac{1}{2}$ and 4. If the third root is c, find $b^2 - 4ac$.

A. $-\frac{21}{2}$ B. 21 C. 25 D. 201 E. 217

22. From a point P on the hyperbola $x^2 - y^2 + 4y - 14 = 0$, tangents are drawn to the circle centered at the origin with radius 1. What is the minimum length of one of these tangents?

A. 3 B. $\sqrt{11}$ C. $2\sqrt{3}$ D. $\sqrt{13}$ E. 4

23. Find the sum of all values of x in the interval $0 \le x < \pi$ that are solutions to the equation: $4\sin 3x \cos^2 x - 2\sin 3x = \cos 2x$.

A. π B. 2π C. 3π D. 4π E. 5π

24. A pyramid is drawn with its base in the shape of an equilateral triangle with side length 12 and with its remaining edges of length $2\sqrt{15}$. If S represents the total surface area, and V represents the volume of the pyramid, then the value of $\frac{S}{V}$ lies in which of the following intervals?

A. [1, 3) B. [3, 5) C. [5, 7) D. [7, 9) E. none of these

25. Given triangle ABC with AC = 18 and point D on AC so that AD = 5. Perpendiculars drawn from D to AB and BC have lengths 4 and 5 respectively. Find the perimeter of triangle ABC.

A. $\frac{18906}{49}$ B. 40 C. $\frac{563}{14}$ D. 42 E. $\frac{360}{7}$

Tie Breakers:

TB1: Find the remainder when 2007! is divided by 28.

TB2: Find the largest integer value of n such that $\sum_{i=-5}^{n}(2i-1) \le 2007$.

TB3: If f(x+1) = x^2 – 3x + 2 and g(x+1) = $\frac{\sqrt{17+x}}{2}$, find $g^{-1}\left[f^{-1}\left(-\frac{1}{4}\right)\right]$.